

Solutions to Maths workbook – 1 | Trigonometric Identities and Equations

Level - 1	Daily Tutorial Sheet - 2
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16. Consider L.H.S. : $[\cos A - \cos B] - \cos C$

$$\begin{aligned}
 &= \left[-2 \sin \frac{A+B}{2} \cdot \sin \frac{A-B}{2} \right] - \cos C = \left[-2 \cos \frac{C}{2} \cdot \sin \frac{A-B}{2} \right] - \left[2 \cos^2 \frac{C}{2} - 1 \right] \\
 &= -2 \cos \frac{C}{2} \left[\sin \frac{A-B}{2} + \cos \frac{C}{2} \right] + 1 = -2 \cos \frac{C}{2} \left[\sin \frac{A-B}{2} + \sin \frac{A+B}{2} \right] + 1 \\
 &= 1 - 4 \sin \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2} = R.H.S.
 \end{aligned}$$

17. $\frac{1}{\cos\left(\frac{\pi}{4} + \theta\right) \cos\left(\frac{\pi}{4} - \theta\right)} = \frac{1}{\cos^2 \frac{\pi}{4} - \sin^2 \theta} = \frac{2}{1 - 2 \sin^2 \theta} = \frac{2}{\cos 2\theta} = 2 \sec 2\theta$ Hence proved

18. $\sqrt{2 + \sqrt{2 + 2 \cos 4\theta}} = \sqrt{2 + \sqrt{2(1 + \cos 4\theta)}} = \sqrt{2 + \sqrt{4 \cos^2 2\theta}} \Rightarrow \sqrt{2 + 2 \cos 2\theta} = \sqrt{2 \cdot 2 \cdot \cos^2 \theta} = 2 \cos \theta$

Note : $\sqrt{4 \cos^2 2\theta} = |2 \cos 2\theta|$ and $\theta \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \Rightarrow \cos 2\theta \geq 0$ and hence, we get $2 \cos 2\theta$

19. Use : $\sin \theta \cdot \sin (60 - \theta) \cdot \sin (60 + \theta) = \frac{1}{4} \sin 3\theta$ **20.** Use : $\sin \theta \cdot \sin (60 - \theta) \cdot \sin (60 + \theta) = \frac{1}{4} \sin 3\theta$

21. $\sqrt{2} [\cos (45 + 15)] = \sqrt{2} (\cos 60) = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$ **22.** $\frac{1 + \tan 9^\circ}{1 - \tan 9^\circ} = \tan(45^\circ + 9^\circ) = \tan 54^\circ$

23. $-4 \sin 18^\circ \sin 54^\circ = -4 \left(\frac{\sqrt{5} - 1}{4} \right) \left(\frac{\sqrt{5} + 1}{4} \right) = -1$
($\sin 234 = -\sin 54$)

24. $\sec \theta = \sqrt{2} \Rightarrow \theta = \frac{7\pi}{4}$ as $\theta \in \text{IV}^{\text{th}}$ quadrant

hence $\tan \theta = -1$, $\operatorname{cosec} \theta = -\sqrt{2} \Rightarrow \frac{1 + \tan \theta + \operatorname{cosec} \theta}{1 + \sec \theta - \operatorname{cosec} \theta} = \frac{-\sqrt{2}}{1 + 2\sqrt{2}}$

25.(ABC) $\frac{\sin 3x}{\sin x} = \frac{3 \sin x - 4 \sin^3 x}{\sin x} \Rightarrow f(x) = 3 - 4 \sin^2 x$ ($\because \sin x \neq 0$)

Now, $0 < \sin^2 x \leq 1$ [$\sin x \neq 0, x \neq n\pi$]
 $-1 \leq -\sin^2 x < 0 \Rightarrow -4 \leq -4 \sin^2 x < 0$
 $\Rightarrow 3 - 4 \leq 3 - 4 \sin^2 x < 3 \Rightarrow -1 \leq f(x) < 3$

26.(A) $1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2 \cos(\pi/4 - \theta) = 1 + \frac{1}{\sqrt{2}} (\sin \theta + \cos \theta) + \sqrt{2} (\sin \theta + \cos \theta)$
 $= 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right) (\sin \theta + \cos \theta) = 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right) \sqrt{2} \cos(\theta - \pi/4)$

$$\leq 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2} \right) \cdot \sqrt{2} = 4 \quad \left[\because \text{max value of } \cos \left(\theta - \frac{\pi}{4} \right) = 1 \right]$$

27.(C) $\cos 2\theta + \cos \theta = 2\cos^2 \theta - 1 + \cos \theta = -1 + 2 \left(\cos^2 \theta + \frac{1}{2} \cos \theta \right)$

$$= -1 + 2 \left[\cos^2 \theta + \frac{1}{2} \cos \theta + \frac{1}{16} - \frac{1}{16} \right] = -1 + 2 \left[\cos \theta + \frac{1}{4} \right]^2 - \frac{1}{8} = -\frac{9}{8} + 2 \left(\cos \theta + \frac{1}{4} \right)^2 \geq -\frac{9}{8}$$

Hence min. value is $-\frac{9}{8}$.

28.(D) $A = \cos^2 \theta + \sin^4 \theta \Rightarrow A = \cos^2 \theta + \sin^2 \theta \cdot \sin^2 \theta$
 $A \leq \sin^2 \theta + \cos^2 \theta \quad [\sin^2 \theta \leq 1] \Rightarrow A \leq 1$ (i)

Again, $\cos^2 \theta + \sin^4 \theta = 1 - \sin^2 \theta + \sin^4 \theta = \left(\sin^2 \theta - \frac{1}{2} \right)^2 + \frac{3}{4} \geq \frac{3}{4}$ (ii)

From (i) and (ii) we get $\frac{3}{4} \leq A \leq 1$

Also, note that $\left(\sin^2 \theta - \frac{1}{2} \right)^2 + \frac{3}{4} \leq 1$

Because $\left(y - \frac{1}{2} \right)^2 + \frac{3}{4} \in \left[\frac{3}{4}, 1 \right] \quad \forall y \in [0, 1]$

29. $(\cos 2A + \cos 2B) + \cos 2C = 2 \cos(A+B) \cdot \cos(A-B) + 2\cos^2 C - 1$
 $= -2 \cos C [\cos(A-B) - \cos C] - 1 = -2 \cos C [\cos(A-B) + \cos(A+B)] - 1 = -2 \cos C [2 \cos A \cdot \cos B] - 1$
 $= -1 - 4 \cos A \cdot \cos B \cdot \cos C = \text{R.H.S.}$

30. $(\cos 2A - \cos 2B) - \cos 2C = [-2 \sin(A+B) \cdot \sin(A-B)] - \cos 2C$
 $= [-2 \sin C \cdot \sin(A-B)] - [1 - 2\sin^2 C]$
 $= [-2 \sin C \cdot \sin(A-B)] + 2\sin^2 C - 1 = -2 \sin C [\sin(A-B) - \sin C] - 1$
 $= -2 \sin C [\sin(A-B) - \sin(A+B)] - 1 = -2 \sin C [-2 \sin B \cdot \cos A] - 1 = -1 + 4 \cos A \sin B \sin C$